

Course's Name analog  
communication  
Course's Number :  
Exam's Period : 1 hours  
Questions' Number :  
Total Mark : 30  
Pages' Number :

Palestine Technical University - Kadoorie

Instructor's Name : Mahmoud Ahmad

Student's Name: .....  
Student's Number: .....  
Section's Number: .....  
Exam's Date : 20/06/2013  
Form :

second.....Exam

second.....Semester 2012/2013

Analog Comm  
2nd Exam

Q1) The modulated wave for a PM signal is given by

$$S(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

a) Write the expression for the instant frequency of  $s(t)$

b) If the modulating signal is as shown below and the phase sensitivity  $K_p = 10\pi$  and the carrier frequency is  $f_c = 100\text{MHz}$

i) Draw the phase modulated wave (1)

ii) Find the maximum and minimum frequency of the modulated wave (2)

c) i) Write the expression for frequency modulated wave and write the instant frequency for this wave (1)

ii) Draw the FM modulated for the same  $m(t)$  (1)

iii) If  $K_f = 10^5 \text{ Hz/v}$  find the maximum and minimum frequency of the modulated wave (2)

$$a) f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

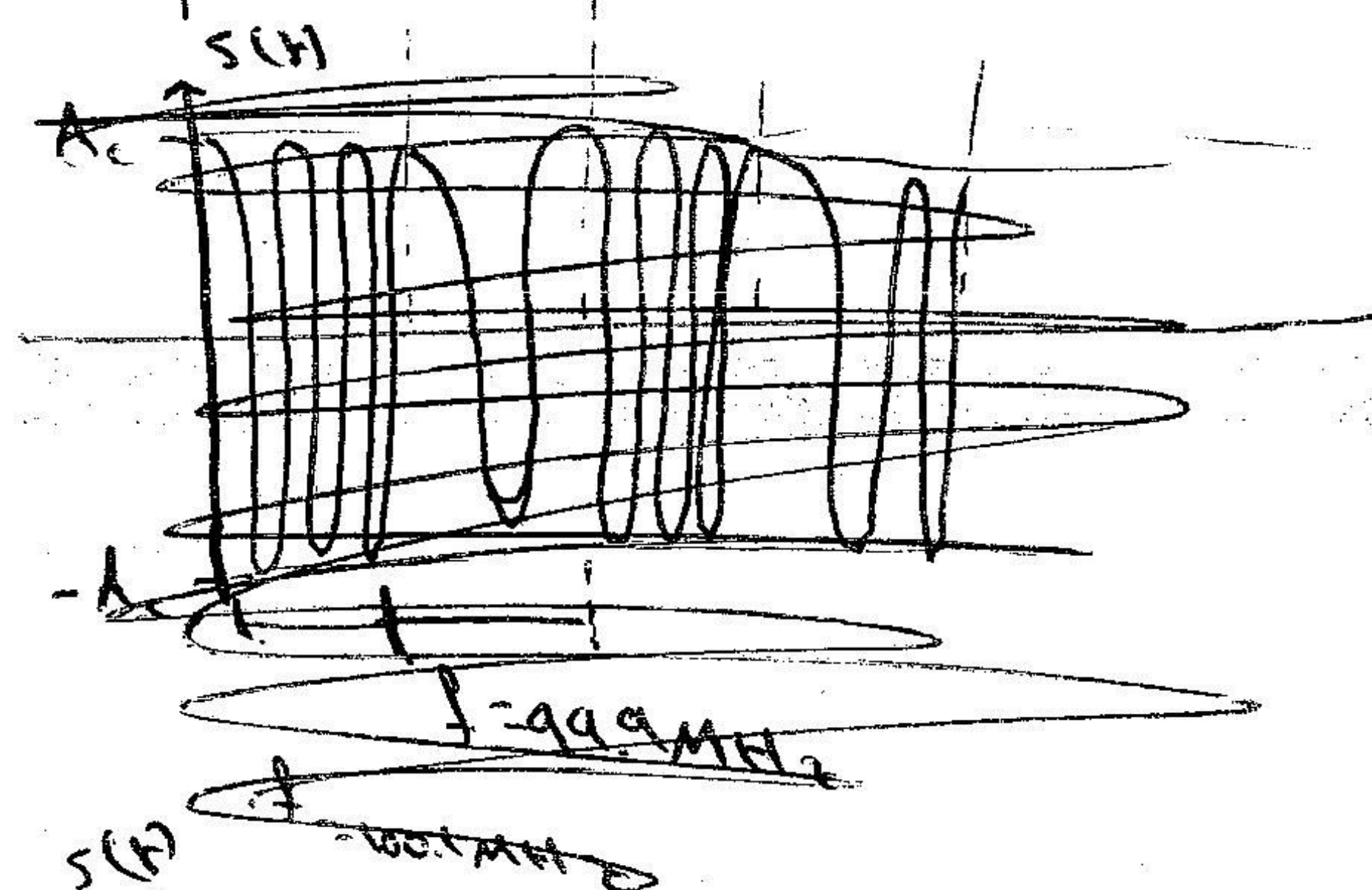
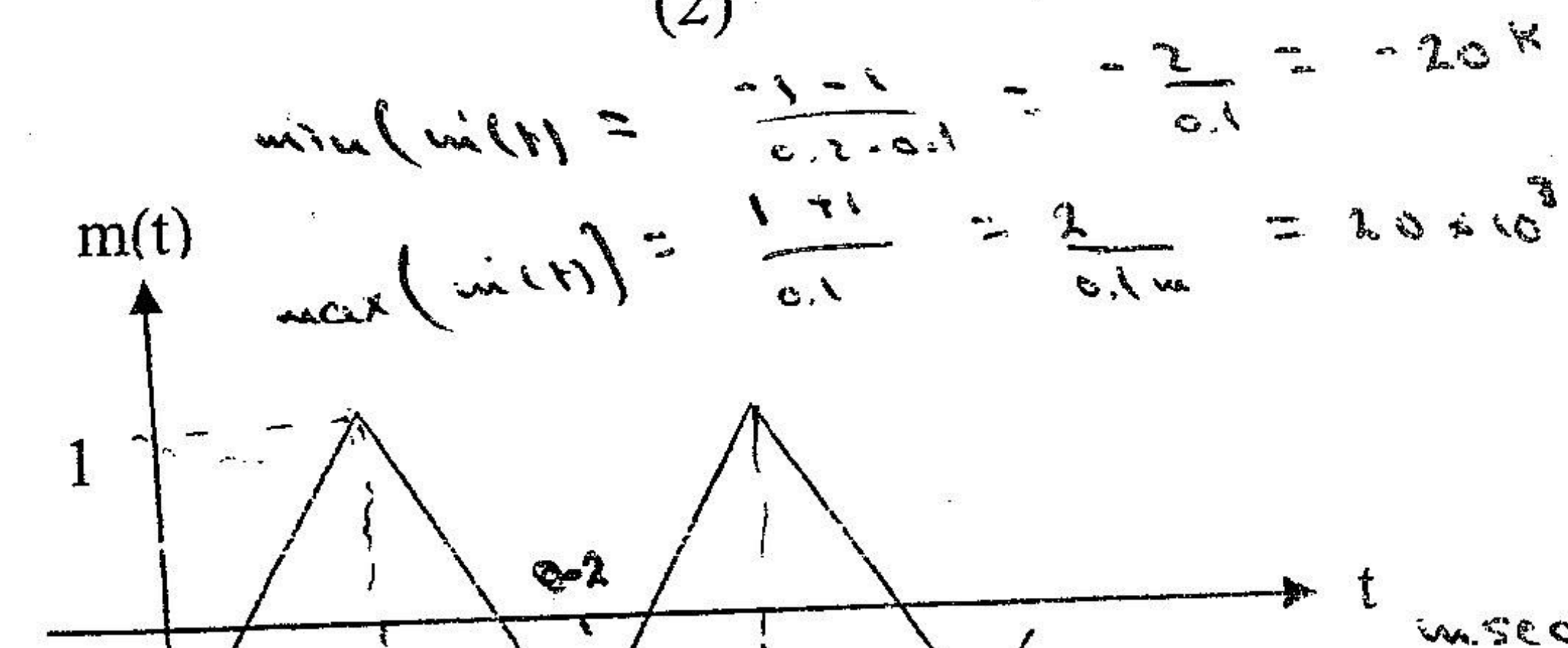
$$= f_c + \frac{K_p}{2\pi} m(t)$$

$$f_{i(\text{max})} = 100 \times 10^6 + \frac{10\pi}{2\pi} \times 20 \times 10^3$$

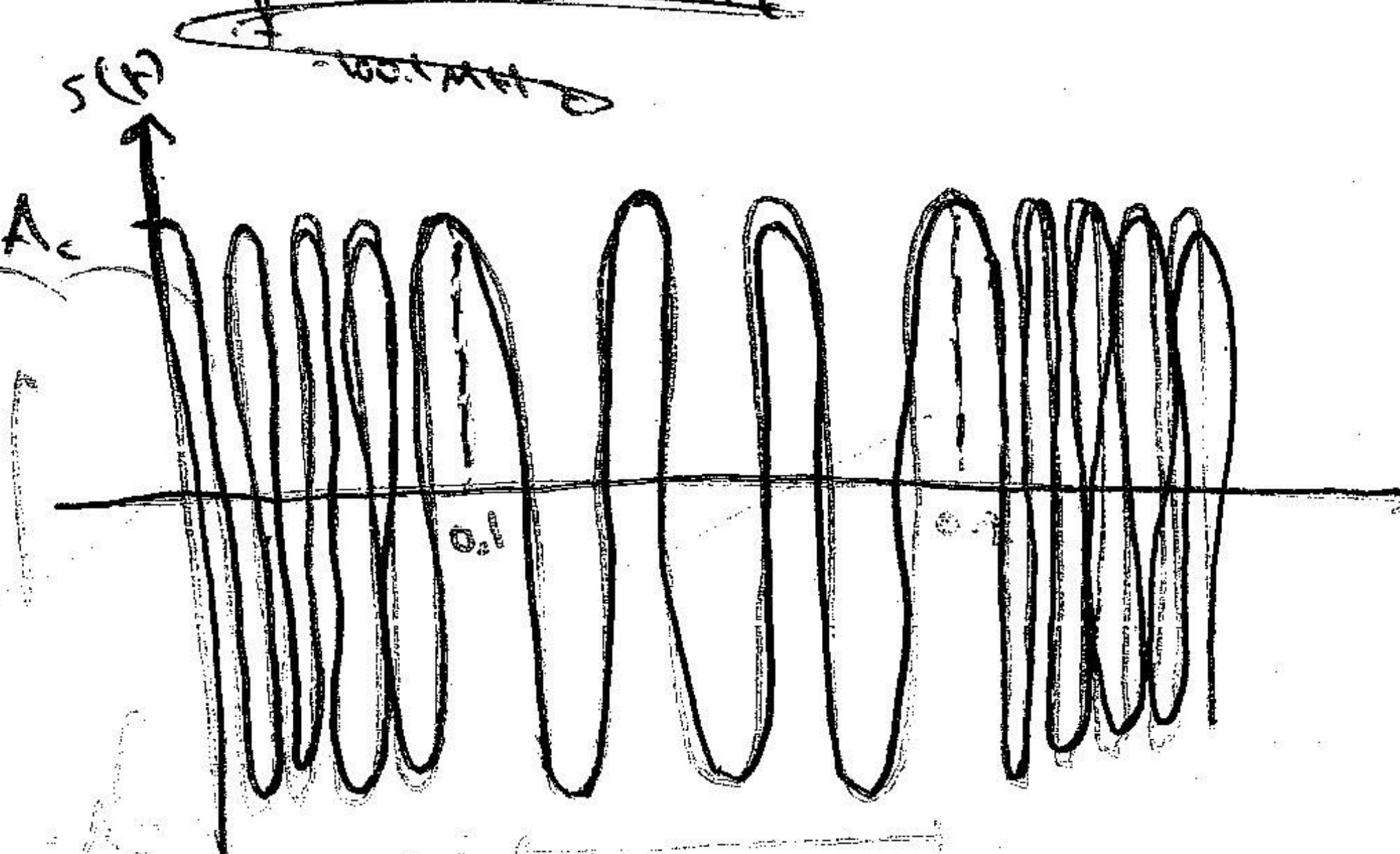
$$= 100.1 \text{ MHz}$$

$$f_{i(\text{min})} = 100 \times 10^6 + \frac{10\pi}{2\pi} \times -20 \times 10^3$$

$$= 99.9 \text{ MHz}$$



(b)



اتصالات تماثلية  
تخضع اتصالات  
الامتصاص الثاني

مع الرفع بواسطة م. من أبو عيسى

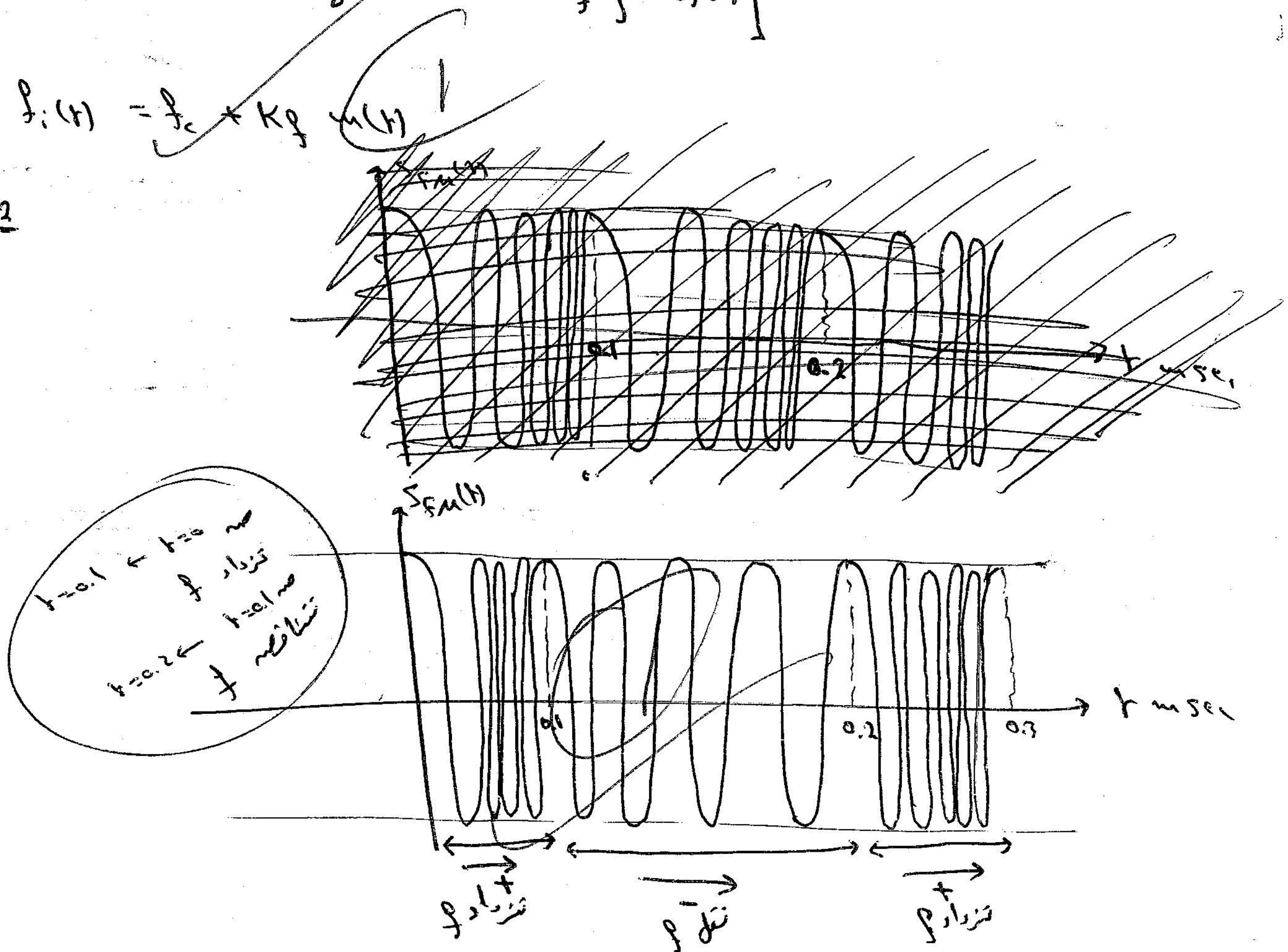


c)

$$s_{FM}(t) = A_c \cos \left[ 2\pi f_c t + 2\pi K_f \int m(t) dt \right]$$

$$f_i(t) = f_c + K_f m(t)$$

2



$$K_f = 10^5 \text{ Hz/V}$$

$$f_{\max} = f_c + 10^5 m(t)_{\max}$$

$$= 100 \text{ MHz} + 10^5 \times 1 = 1.1 \times 10^6 \text{ Hz} = 100.1 \text{ MHz}$$

$$f_{\min} = f_c + 10^5 m(t)_{\min}$$

$$= 100 \text{ MHz} + 10^5 (-1) = 99.9 \text{ MHz}$$



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Q2) Consider the multi-tone modulating wave  
 $m(t) = A_m \cos(2\pi f_a t) \cos(2\pi f_b t)$

- a) Find the corresponding PM & FM modulated waves. Assume phase sensitivity of  $K_p$  rad/v and frequency sensitivity of  $K_f$  Hz/v what is modulation index for the two cases (2)  
b) Derive the NBFM modulated signal for the modulating wave  $m(t)$  (3)  
c) Find and draw the spectrum of the NBFM signal derived in b (3)

$$a) S_{PM}(t) = A_c \cos[2\pi f_c t + K_p A_m \cos(2\pi f_a t) \cos(2\pi f_b t)]$$

$$S_{FM}(t) = A_c \cos[2\pi f_c t + K_f A_m 2\pi \int \cos(2\pi f_a t) \cos(2\pi f_b t) dt]$$

$$\beta_{PM} = K_p A_m$$

$$\beta_{FM} = \frac{K_f A_m}{2}$$

$$m(t) = \frac{A_m}{2} [\cos 2\pi(f_a - f_b)t + \cos 2\pi(f_a + f_b)t]$$

for FM

$$f_i(t) = f_c + K_f m(t)$$

$$\theta(t) = 2\pi f_c t + K_f 2\pi \left[ \frac{\sin 2\pi(f_a - f_b)t}{2\pi(f_a - f_b)} + \frac{\sin 2\pi(f_a + f_b)t}{2\pi(f_a + f_b)} \right] \frac{A_m}{2}$$

$$S_{FM}(t) = \cos \left[ 2\pi f_c t + \frac{K_f A_m}{2(f_a - f_b)} \sin 2\pi(f_a - f_b)t + \frac{K_f A_m}{2(f_a + f_b)} \sin 2\pi(f_a + f_b)t \right]$$

PMs

$$\theta(t) = 2\pi f_c t + \frac{K_p A_m}{2} [\cos 2\pi(f_a - f_b)t + \cos 2\pi(f_a + f_b)t]$$

$$S_{PM}(t) = \cos \left[ 2\pi f_c t + \frac{K_p A_m}{2} \cos 2\pi(f_a - f_b)t + \frac{K_p A_m}{2} \cos 2\pi(f_a + f_b)t \right]$$



$$b) S_{FM}(t) = \cos \left[ 2\pi f_c t + \frac{K_f A_m}{2(f_a - f_b)} \sin 2\pi(f_a - f_b)t + \frac{K_f A_m}{2(f_a + f_b)} \sin 2\pi(f_a + f_b)t \right]$$

$$S_{FM}(t) = \cos \left[ 2\pi f_c t + 2\pi K_f A_m \int \cos 2\pi f_a t \cos 2\pi f_b t \right]$$

$$= \cos 2\pi f_c t \cos \left[ 2\pi K_f A_m \int \cos 2\pi f_a t \cos 2\pi f_b t \right]$$

$$- \sin 2\pi f_c t \sin \left[ 2\pi K_f A_m \int \cos 2\pi f_a t \cos 2\pi f_b t \right]$$

since it NBFM

$$\cos \left[ 2\pi K_f A_m \int \cos 2\pi f_a t \cos 2\pi f_b t \right] \approx 1$$

$$\sin \left[ 2\pi K_f A_m \int \cos 2\pi f_a t \cos 2\pi f_b t \right] \approx 2\pi K_f A_m \int \cos 2\pi f_a t \cos 2\pi f_b t dt$$

$$S_{FM} = \cos 2\pi f_c t - 2\pi K_f A_m \sin(2\pi f_c t) \int \cos 2\pi f_a t \cos 2\pi f_b t dt$$

$$\int \cos 2\pi f_a t \cos 2\pi f_b t dt = \left( \frac{\sin 2\pi(f_a - f_b)t}{2\pi(f_a - f_b)} + \frac{\sin 2\pi(f_a + f_b)t}{2\pi(f_a + f_b)} \right) \cdot \frac{1}{2}$$

$$S_{FM} = \cos 2\pi f_c t - K_f A_m \left( \frac{\sin 2\pi f_c t \sin 2\pi(f_a - f_b)t}{2(f_a - f_b)} - \frac{\sin 2\pi f_c t \sin 2\pi(f_a + f_b)t}{2(f_a + f_b)} \right)$$

$$= \cos 2\pi f_c t - \frac{A_m K_f}{4(f_a - f_b)} \left[ \cos 2\pi(f_c - f_a + f_b)t - \cos 2\pi(f_c + f_a - f_b)t \right]$$

$$- \frac{K_f A_m}{4(f_a + f_b)} \left[ \cos 2\pi(f_c - f_a - f_b)t - \cos 2\pi(f_c + f_a + f_b)t \right]$$

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Q3) A carrier wave of frequency  $100\text{MHz}$  is frequency modulated by a sinusoidal wave of amplitude  $A_m$  of  $20\text{V}$  and frequency of  $100\text{KHz}$ . The frequency sensitivity of the modulator is  $30\text{KHz/V}$

- Write the expression for the FM modulated wave (1)
- Find the approximate band width of the FM wave using Carson rule (1.5)
- Find the band width of the FM wave using 10% rule (1.5)
- Find the band width of the FM wave using 1% rule (1)
- What is the band width of the FM wave if the amplitude of the modulated wave is decreased to 1 volt draw the spectrum of the modulated wave in this case (3)

$A_m = 20 \rightarrow 1$

$$a) S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$= \cos[200\pi \times 10^6 t + 6 \sin(200\pi \times 10^3 t)]$$

$$\beta = \frac{K_f A_m}{f_m} = \frac{30 \times 10^3 \times 20}{100 \times 10^3} = 6$$

$$b) BW = 2f_m(\beta + 1) = 2 \times 100 \times 10^3 (7) = 1400 \text{ KHz}$$

$$c) m_{max} = 7$$

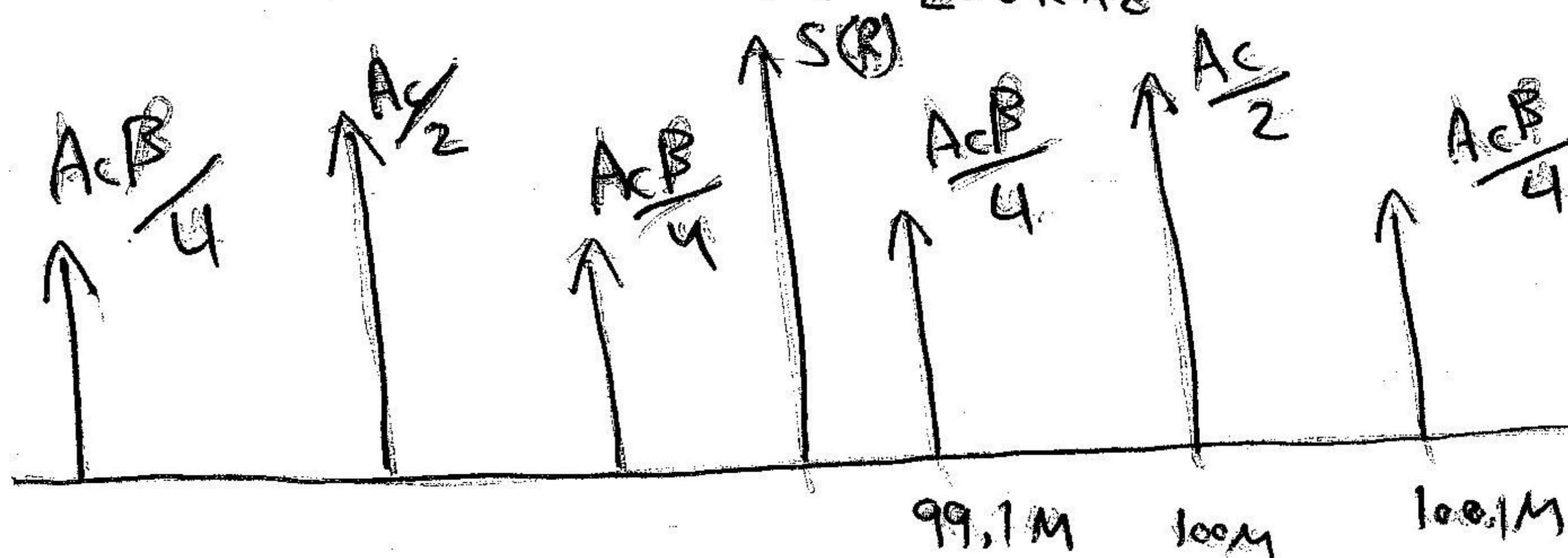
$$BW = 2 \times 7 \times f_m = 14 \times 100 \text{ K} = 1400 \text{ KHz}$$

$$d) m_{max} = 9$$

$$BW = 2 \times 9 \times 100 \text{ K} = 1800 \text{ KHz}$$

$$e) \beta = \frac{K_f A_m}{f_m} = \frac{30 \times 1}{100 \times 10^3} = 0.3 \quad \text{NBFM}$$

$$BW = 2 \times f_m = 2 \times 100 \text{ K} = 200 \text{ KHz}$$





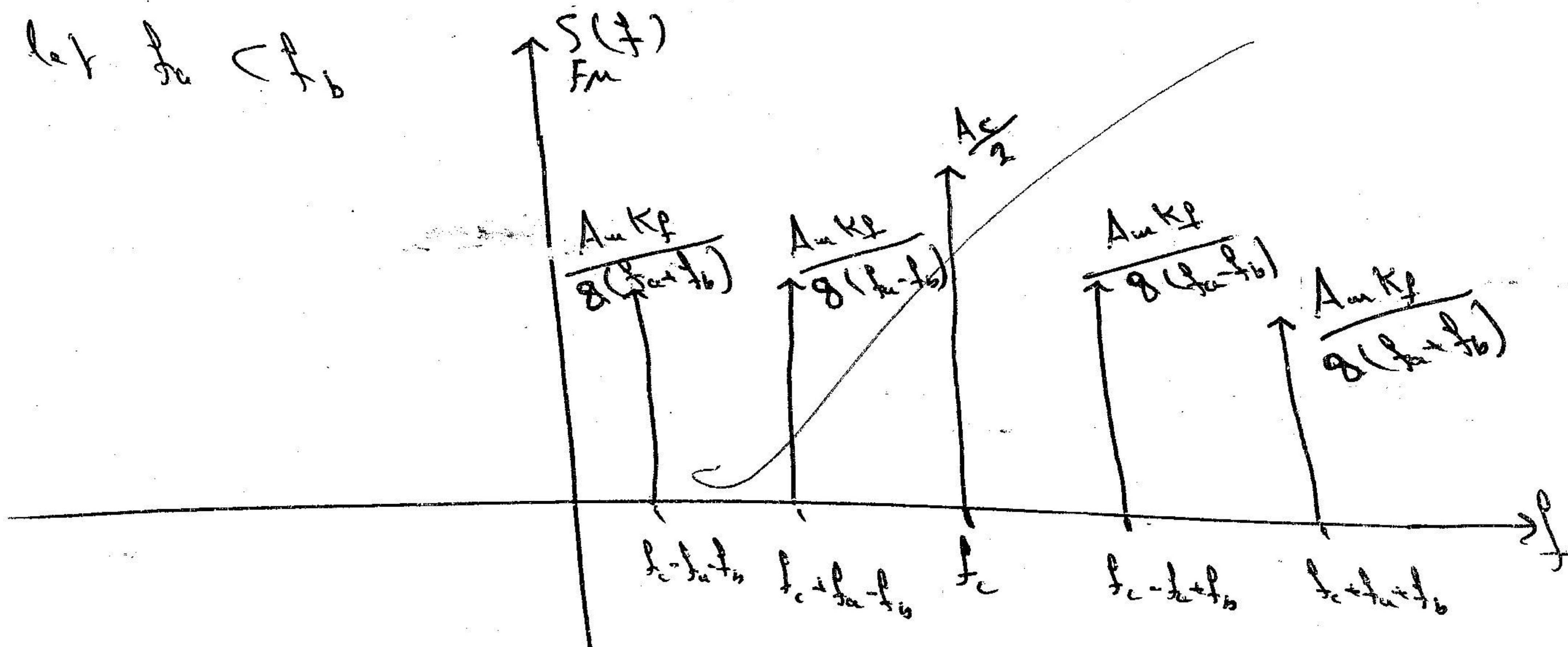
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سوال 2

$$S_{FM}(t) = \cos 2\pi f_c t - \frac{A_m K_f}{4\pi(f_c - f_a - f_b)} \left[ \cos 2\pi(f_c - f_a + f_b)t + \cos 2\pi(f_c + f_a - f_b)t \right]$$

$$- \frac{A_m K_f}{4\pi(f_a + f_b)} \left[ \cos 2\pi(f_c - f_a - f_b)t + \cos 2\pi(f_c + f_a + f_b)t \right]$$

let  $f_a < f_b$



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Q4) The sinusoidal modulating wave  $m(t) = A_m \cos(2\pi f_m t)$  is the input of a phase modulator with phase sensitivity  $K_p$ . the unmodulated carrier has a frequency  $f_c$  and amplitude  $A_c$ . Determine the spectrum of the resulting PM wave assuming the maximum phase deviation does not exceed 0.3

(6)

$\beta_p$

$$S(t) = A_c \cos \left[ 2\pi f_c t + K_p A_m \cos 2\pi f_m t \right]$$

$\beta_p < 0.3$

NBPM

$\beta_p \cos 2\pi f_m t$

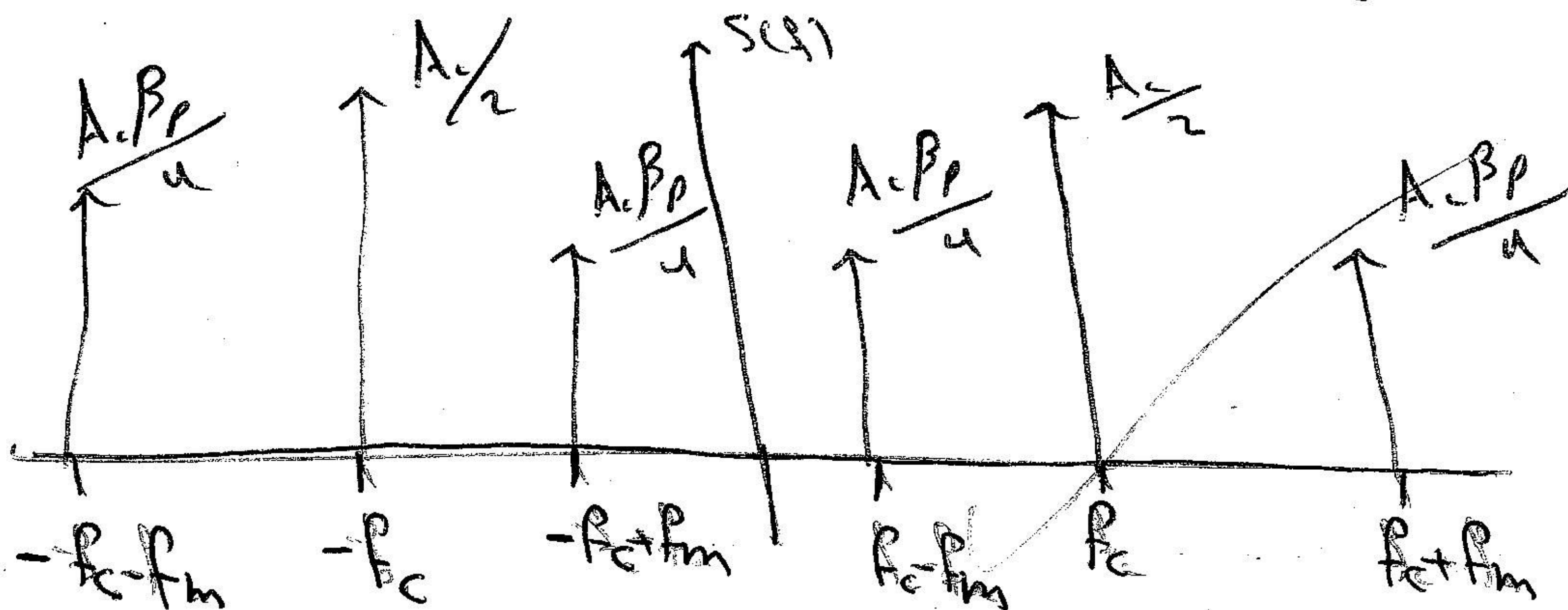
$$S(t) = A_c \cos[2\pi f_c t] \cos[\beta_p \cos 2\pi f_m t] - A_c \sin 2\pi f_c t \sin[\beta_p \cos 2\pi f_m t]$$

$$\beta_p < 0.3 \Rightarrow \beta_p \cos 2\pi f_m t < 0.3$$

$$\theta \ll 1 \Rightarrow \sin \theta \approx \theta$$

$$\therefore S(t) = A_c \cos 2\pi f_c t - A_c \beta_p \sin 2\pi f_c t \cos 2\pi f_m t$$

$$= A_c \cos 2\pi f_c t - \frac{A_c \beta_p}{2} \left[ \sin 2\pi (f_c - f_m) t + \sin 2\pi (f_c + f_m) t \right]$$





# Table A4.1

## Table of Bessel Functions

$n \backslash x$	$J_n(x)$								
	0.5	1	2	3	4	6	8	10	12
0	0.9385	0.7652	0.2239	-0.2601	-0.3971	0.1506	0.1717	-0.2459	0.0477
1	0.2423	0.4401	0.5767	0.3391	-0.0660	-0.2767	0.2346	0.0435	-0.2234
2	0.0306	0.1149	0.3528	0.4861	0.3641	-0.2429	-0.1130	0.2546	-0.0849
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	-0.2911	0.0584	0.1951
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	-0.1054	-0.2196	0.1825
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	-0.2341	-0.0735
6	—	—	0.0012	0.0114	0.0491	0.2458	0.3376	-0.0145	-0.2437
7	—	—	0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	-0.1703
8	—	—	—	0.0005	0.0040	0.0565	0.2235	0.3179	0.0451
9	—	—	—	0.0001	0.0009	0.0212	0.1263	0.2919	0.2304
10	—	—	—	—	0.0002	0.0070	0.0608	0.2075	0.3005
11	—	—	—	—	—	0.0020	0.0256	0.1231	0.2704
12	—	—	—	—	—	0.0005	0.0096	0.0634	0.1953
13	—	—	—	—	—	0.0001	0.0033	0.0290	0.1201
14	—	—	—	—	—	—	0.0010	0.0120	0.0650



Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) = aG_1(f) + bG_2(f)$ where $a$ and $b$ are constants
2. Time scaling	$g(at) = \frac{1}{ a } G\left(\frac{f}{a}\right)$ where $a$ is a constant
3. Duality	If $g(t) = G(f)$ , then $G(t) = g(-f)$
4. Time shifting	$g(t - t_0) = G(f) \exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t) g(t) = G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t) dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f) df$
8. Differentiation in the time domain	$\frac{d}{dt} g(t) = j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{\infty} g(\tau) d\tau = \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
10. Conjugate functions	If $g(t) = G(f)$ , then $g^*(t) = G^*(-f)$
11. Multiplication in the time domain	$g_1(t) g_2(t) = \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda) d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau) d\tau = G_1(f) G_2(f)$

## 640 Appendix 6

Table A6.4 Trigonometric Identities

$\exp(\pm j\theta) = \cos \theta \pm j \sin \theta$
$\cos \theta = \frac{1}{2} [\exp(j\theta) + \exp(-j\theta)]$
$\sin \theta = \frac{1}{2j} [\exp(j\theta) - \exp(-j\theta)]$
$\sin^2 \theta + \cos^2 \theta = 1$
$\cos^2 \theta - \sin^2 \theta = \cos(2\theta)$
$\cos^2 \theta = \frac{1}{2} [1 + \cos(2\theta)]$
$\sin^2 \theta = \frac{1}{2} [1 - \cos(2\theta)]$
$2 \sin \theta \cos \theta = \sin(2\theta)$
$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$
$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

The Fourier representation of  $sq(\theta(t))$

$$sq(\theta(t)) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1} \cos[2\pi(2k-1)f_c t + (2k-1)\phi(t)]$$

For a square periodic function with period  $T_c$  The

Fourier representation of  $p(t)$  is

$$p(t) = \sum_{n=1}^{\infty} 2 \operatorname{sinc}(n/2) \cos 2\pi n f_c t = \frac{4}{\pi} \cos 2\pi f_c t - \frac{4}{3\pi} \cos 6\pi f_c t + \dots$$